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# Precision Observables in the MSSM: Leading Electroweak Two-loop Corrections

S. HEINEMEYER\* AND G. WEIGLEIN†

\**HET, Brookhaven Natl. Lab., Upton NY 11973, USA*

†*CERN, TH Division, CH-1211 Geneva 23, Switzerland*

The leading electroweak MSSM two-loop corrections to the  $\rho$ -parameter are calculated. They are obtained by evaluating the two-loop self-energies of the  $Z$  and the  $W$  boson at  $\mathcal{O}(G_F^2 m_t^4)$  in the limit of heavy scalar quarks. A very compact expression is derived, depending on the ratio of the  $\mathcal{CP}$ -odd Higgs boson mass,  $M_A$ , and the top quark mass,  $m_t$ . Expressions for the limiting cases  $M_A \gg m_t$  and  $M_A \ll m_t$  are also given. The decoupling of the non-SM contribution in the limit  $M_A \rightarrow \infty$  is verified at the two-loop level. The numerical effect of the leading electroweak MSSM two-loop corrections is analyzed in comparison with the leading corrections of  $\mathcal{O}(G_F^2 m_t^4)$  in the SM and with the  $\mathcal{O}(\alpha\alpha_s)$  corrections in the MSSM.

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# 1 Introduction

Theories based on Supersymmetry (SUSY) [1] are widely considered as the theoretically most appealing extension of the Standard Model (SM). They predict the existence of scalar partners  $\tilde{f}_L, \tilde{f}_R$  to each SM chiral fermion, and spin-1/2 partners to the gauge bosons and to the scalar Higgs bosons. So far, the direct search for SUSY particles has not been successful. One can only set lower bounds of  $\mathcal{O}(100)$  GeV on their masses [2]. Contrary to the SM, two Higgs doublets are required in the Minimal Supersymmetric Standard Model (MSSM) resulting in five physical Higgs bosons [3]. The direct search resulted in lower limits of about 90 GeV for the neutral Higgs bosons [4].

An alternative way to probe SUSY is to search for the virtual effects of the additional particles via precision observables. The most prominent role in this respect plays the  $\rho$ -parameter [5]. The leading radiative corrections to the  $\rho$ -parameter,  $\Delta\rho$ , constitute the leading process-independent corrections to many electroweak precision observables, such as the  $W$  boson mass,  $M_W$ , and the effective leptonic weak mixing angle,  $\sin^2 \theta_{\text{eff}}$ . Within the MSSM the full one-loop corrections to  $M_W$  and  $\sin^2 \theta_{\text{eff}}$  have been calculated already several years ago [6,7]. More recently also the leading two-loop corrections of  $\mathcal{O}(\alpha\alpha_s)$  to the quark and scalar quark loops for  $\Delta\rho$  and  $M_W$  have been obtained [8,9]. Contrary to the SM case, these two-loop corrections turned out to increase the one-loop contributions, leading to an enhancement of the latter of up to 35% [8].

We summarize here the result for the leading two-loop corrections to  $\Delta\rho$  at  $\mathcal{O}(G_F^2 m_t^4)$  [10]. For a large SUSY scale,  $M_{SUSY} \gg M_Z$ , the SUSY contributions decouple from physical observables. This has been verified with existing results at the one-loop [11] and at the two-loop level [8,10]. Therefore, in the case of large  $M_{SUSY}$  the leading electroweak two-loop corrections in the MSSM are obtained in the limit where besides the SM particles only the two Higgs doublets needed in the MSSM are active. We derive the result for the  $\mathcal{O}(G_F^2 m_t^4)$  [10] corrections in this case and provide a compact analytical formula for it, depending on the  $\mathcal{CP}$ -odd Higgs boson mass,  $M_A$ , and the top quark mass,  $m_t$ . Furthermore, we present formulas for the limiting cases  $M_A \gg m_t$  (i.e. the SM limit) and  $M_A \ll m_t$ . The numerical effect of the  $\mathcal{O}(G_F^2 m_t^4)$  corrections is compared with the corresponding SM result [12] and the gluon-exchange correction of  $\mathcal{O}(\alpha\alpha_s)$  in the MSSM.

## 2 Calculation of the $\mathcal{O}(G_F^2 m_t^4)$ corrections

### 2.1 $\Delta\rho$ and the Higgs sector

The quantity  $\Delta\rho$ ,

$$\Delta\rho = \frac{\Sigma_Z(0)}{M_Z^2} - \frac{\Sigma_W(0)}{M_W^2}, \quad (1)$$

parameterizes the leading universal corrections to the electroweak precision observables induced by the mass splitting between fields in an isospin doublet [5].  $\Sigma_{Z,W}(0)$  denote the transverse parts of the unrenormalized  $Z$  and  $W$  boson self-energies at zero momentum transfer, respectively. The shifts induced by  $\Delta\rho$  in the prediction for the  $W$  boson mass,  $M_W$ , and the effective leptonic weak mixing angle,  $\sin^2 \theta_{\text{eff}}$ , are approximately given by

$$\delta M_W \approx \frac{M_W}{2} \frac{c_W^2}{c_W^2 - s_W^2} \Delta\rho, \quad \delta \sin^2 \theta_{\text{eff}} \approx -\frac{c_W^2 s_W^2}{c_W^2 - s_W^2} \Delta\rho. \quad (2)$$

Contrary to the SM, in the MSSM two Higgs doublets are required [3]. At the tree-level, the Higgs sector can be described in terms of two independent parameters (besides  $g$  and  $g'$ ): the ratio of the two vacuum expectation values,  $\tan\beta = v_2/v_1$ , and  $M_A$ , the mass of the  $\mathcal{CP}$ -odd  $A$  boson. The diagonalization of the bilinear part of the Higgs potential, i.e. the Higgs mass matrices, is performed via orthogonal transformations with the angle  $\alpha$  for the  $\mathcal{CP}$ -even part and with the angle  $\beta$  for the  $\mathcal{CP}$ -odd and the charged part. The mixing angle  $\alpha$  is determined at lowest order through

$$\tan 2\alpha = \tan 2\beta \frac{M_A^2 + M_Z^2}{M_A^2 - M_Z^2}; \quad -\frac{\pi}{2} < \alpha < 0. \quad (3)$$

One gets the following Higgs spectrum:

$$\begin{aligned} 2 \text{ neutral bosons, } \mathcal{CP} = +1 & : h^0, H^0 \\ 1 \text{ neutral boson, } \mathcal{CP} = -1 & : A^0 \\ 2 \text{ charged bosons} & : H^+, H^- \\ 3 \text{ unphysical scalars} & : G^0, G^+, G^-. \end{aligned} \quad (4)$$

The tree-level masses, expressed through  $M_Z$ ,  $M_W$  and  $M_A$ , are given by

$$\begin{aligned} m_h^2 &= \frac{1}{2} \left[ M_A^2 + M_Z^2 - \sqrt{(M_A^2 + M_Z^2)^2 - 4M_A^2 M_Z^2 \cos^2 2\beta} \right] \\ m_H^2 &= \frac{1}{2} \left[ M_A^2 + M_Z^2 + \sqrt{(M_A^2 + M_Z^2)^2 - 4M_A^2 M_Z^2 \cos^2 2\beta} \right] \\ m_{H^\pm}^2 &= M_A^2 + M_W^2 \\ m_G^2 &= M_Z^2 \\ m_{G^\pm}^2 &= M_W^2, \end{aligned} \quad (5)$$

where the last two relations, which assign mass parameters to the unphysical scalars  $G^0$  and  $G^\pm$ , are to be understood in the Feynman gauge.

## 2.2 Evaluation of the $\mathcal{O}(G_F^2 m_t^4)$ contributions

In order to calculate the  $\mathcal{O}(G_F^2 m_t^4)$  corrections to  $\Delta\rho$  in the approximation that all superpartners are heavy so that their contribution decouples, the Feynman diagrams generically depicted in Fig. 1 have to be evaluated for the  $Z$  boson ( $V = Z$ ) and the  $W$  boson ( $V = W$ ) self-energy. We have taken into account all possible combinations of the  $t/b$  doublet and the full Higgs sector of the MSSM, see Sect. 2.1.

The two-loop diagrams shown in Fig. 1 have to be supplemented with the corresponding one-loop diagrams with subloop renormalization, depicted generically in Fig. 2. The corresponding insertions for the fermion and Higgs mass counter terms are shown in Fig. 3.

The amplitudes of all Feynman diagrams, shown in Figs. 1–3, have been created with the program *FeynArts2.2* [13], making use of a recently completed model file for the MSSM<sup>1</sup>. The algebraic evaluation and reduction to scalar integrals has been performed with the program *TwoCalc* [14]. (Further details about the evaluations with *FeynArts2.2* and *TwoCalc* can be found in Ref. [15].) As a result we obtained the analytical expression for  $\Delta\rho$  depending on the one-loop functions  $A_0$  and  $B_0$  [16] and on the two-loop function  $T_{134}$  [14,17]. For the further evaluation the analytical expressions for  $A_0$ ,  $B_0$  and  $T_{134}$  have been inserted. In order to derive the leading contributions of  $\mathcal{O}(G_F^2 m_t^4)$  we extracted a prefactor  $h_t^4 \sim G_F^2 m_t^4$ . Its coefficient can be evaluated in the limit where  $M_W$  and  $M_Z$  (and also  $m_b$ ) are set to zero. Furthermore we made use of the mass relations in the MSSM Higgs sector, see eq. (5). In the limit  $M_W, M_Z \rightarrow 0$  they reduce to

$$\begin{aligned} m_h^2 &= 0 \\ m_H^2 &= M_A^2 \\ m_{H^\pm}^2 &= M_A^2 \\ m_G^2 &= 0 \\ m_{G^\pm}^2 &= 0. \end{aligned} \tag{6}$$

In the limit  $M_Z \rightarrow 0$  the relation between the angles  $\alpha$  and  $\beta$ , see eq. (3), becomes very simple,  $\alpha = \beta - \pi/2$ , i.e.  $\sin \alpha = -\cos \beta$ ,  $\cos \alpha = \sin \beta$ . The coefficient of the leading  $\mathcal{O}(G_F^2 m_t^4)$  term thus depends only on the top quark mass,  $m_t$ , the  $\mathcal{CP}$ -odd Higgs boson mass,  $M_A$ , and  $\tan \beta$  (or  $s_\beta = \tan \beta / \sqrt{1 + \tan^2 \beta}$ ).

We explicitly verified the UV-finiteness of our result. As a further consistency check of our method we also recalculated the SM result for the  $\mathcal{O}(G_F^2 m_t^4)$  corrections

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<sup>1</sup> Only the non-SM like counter terms had to be added.

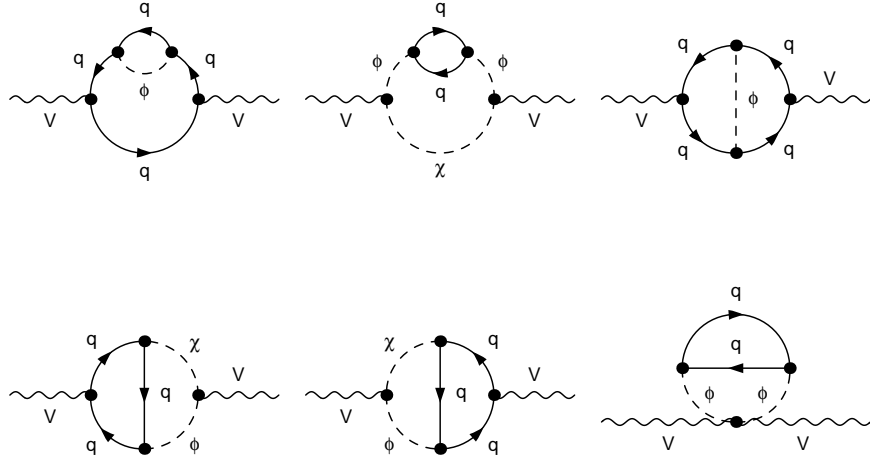


Figure 1: Generic Feynman diagrams for the vector boson self-energies ( $V = \{Z, W\}, q = \{t, b\}, \phi, \chi = \{h, H, A, H^\pm, G, G^\pm\}$ ).

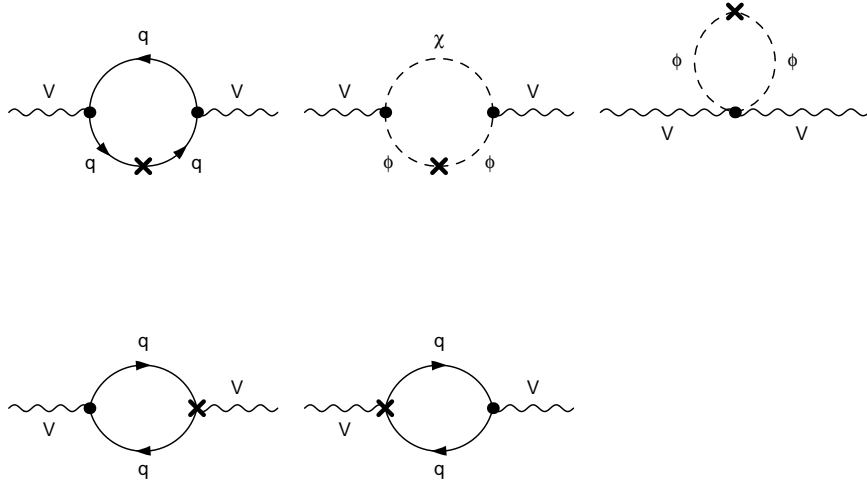


Figure 2: Generic Feynman diagrams for the vector boson self-energies with counter term insertion ( $V = \{Z, W\}, q = \{t, b\}, \phi, \chi = \{h, H, A, H^\pm, G, G^\pm\}$ ).

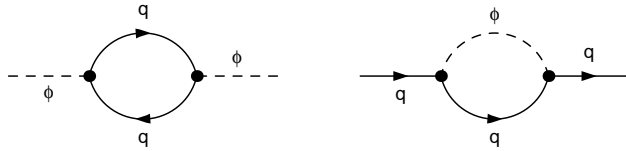


Figure 3: Generic Feynman diagrams for the counter term insertions ( $q = \{t, b\}, \phi = \{h, H, A, H^\pm, G, G^\pm\}$ ).

with arbitrary values of the Higgs boson mass, as given in Ref. [18], and found perfect agreement.

### 3 Analytical results

#### 3.1 The full result

The analytical result obtained as described in Sect. 2.2 can conveniently be expressed in terms of

$$a \equiv \frac{m_t^2}{M_A^2}. \quad (7)$$

The corresponding two-loop contribution to  $\Delta\rho$  then reads:

$$\begin{aligned} \Delta\rho_{1,\text{Higgs}}^{\text{SUSY}} = & 3 \frac{G_F^2}{128 \pi^4} m_t^4 \frac{1 - s_\beta^2}{s_\beta^2 a^2} \times \\ & \left\{ \text{Li}_2 \left( (1 - \sqrt{1 - 4a}) / 2 \right) \frac{8}{\sqrt{1 - 4a}} \Lambda \right. \\ & - 2 \text{Li}_2 \left( 1 - \frac{1}{a} \right) [5 - 14a + 6a^2] \\ & + \log^2(a) \left[ 1 + \frac{2}{\sqrt{1 - 4a}} \Lambda \right] - \log(a) [2 - 20a] \\ & - \log^2 \left( \frac{1 - \sqrt{1 - 4a}}{2} \right) \frac{4}{\sqrt{1 - 4a}} \Lambda \\ & + \log \left( \frac{1 - \sqrt{1 - 4a}}{1 + \sqrt{1 - 4a}} \right) \sqrt{1 - 4a} (1 - 2a) \\ & - \log(|1/a - 1|) (a - 1)^2 \\ & \left. + \pi^2 \left[ \frac{2\sqrt{1 - 4a}}{-3 + 12a} \Lambda + \frac{1}{3} - 2a^2 \frac{s_\beta^2}{1 - s_\beta^2} \right] - 17a + 19 \frac{a^2}{1 - s_\beta^2} \right\}, \quad (8) \end{aligned}$$

with

$$\Lambda = 3 - 13a + 11a^2. \quad (9)$$

In the limit of large  $\tan\beta$  (i.e.  $(1 - s_\beta^2) \ll 1$ ) one obtains

$$\Delta\rho_{1,\text{Higgs}}^{\text{SUSY}} = 3 \frac{G_F^2}{128 \pi^4} m_t^4 \left[ \frac{19}{s_\beta^2} - 2\pi^2 + \mathcal{O}(1 - s_\beta^2) \right]. \quad (10)$$

Thus for large  $\tan\beta$  the SM limit with  $M_H^{\text{SM}} \rightarrow 0$  [12] is reached.

### 3.2 The expansion for large $M_A$

The result for  $\Delta\rho_{1,\text{Higgs}}^{\text{SUSY}}$  in eq. (8) can be expanded for small values of  $a$ , i.e. for large values of  $M_A$ :

$$\begin{aligned} \Delta\rho_{1,\text{Higgs}}^{\text{SUSY}} = & 3 \frac{G_F^2}{128 \pi^4} m_t^4 \times \\ & \left\{ 19 - 2\pi^2 \right. \\ & - \frac{1 - s_\beta^2}{s_\beta^2} \left[ \left( \log^2 a + \frac{\pi^2}{3} \right) (8a + 32a^2 + 132a^3 + 532a^4) \right. \\ & + \log(a) \frac{1}{30} (560a + 2825a^2 + 11394a^3 + 45072a^4) \\ & - \frac{1}{1800} (2800a + 66025a^2 + 300438a^3 + 1265984a^4) \\ & \left. \left. + \mathcal{O}(a^5) \right] \right\}. \end{aligned} \quad (11)$$

In the limit  $a \rightarrow 0$  one obtains

$$\Delta\rho_{1,\text{Higgs}}^{\text{SUSY}} = 3 \frac{G_F^2}{128 \pi^4} m_t^4 [19 - 2\pi^2] + \mathcal{O}(a), \quad (12)$$

i.e. exactly the SM limit for  $M_H^{\text{SM}} \rightarrow 0$  is reached. This constitutes an important consistency check: in the limit  $a \rightarrow 0$  the heavy Higgs bosons decouple from the theory. Thus only the lightest  $\mathcal{CP}$ -even Higgs boson remains, which has in the  $\mathcal{O}(G_F^2 m_t^4)$  approximation the mass  $m_h = 0$ , see eq. (6). This decoupling of the non-SM contributions in the limit where the new scale (i.e. in the present case  $M_A$ ) is made large is explicitly seen here at the two-loop level.

### 3.3 The expansion for small $M_A$

The result for  $\Delta\rho_{1,\text{Higgs}}^{\text{SUSY}}$  in eq. (8) can also be expanded for large values of  $a$ , i.e. for small values of  $M_A$  (with  $\hat{a} = 1/a$ ):

$$\begin{aligned} \Delta\rho_{1,\text{Higgs}}^{\text{SUSY}} = & 3 \frac{G_F^2}{128 \pi^4} m_t^4 \times \\ & \left\{ \log^2(\hat{a}) \hat{a}^2 \left[ -1 + \frac{1}{s_\beta^2} \right] \right. \\ & + \log(\hat{a}) \frac{1 - s_\beta^2}{210 s_\beta^2} [-2100\hat{a} + 350\hat{a}^2 + 504\hat{a}^3 + 341\hat{a}^4] \\ & \left. + \pi^2 \frac{2}{3 s_\beta^2} [-3 + 7\hat{a}(1 - s_\beta^2) - 2\hat{a}^2(1 - s_\beta^2)] \right\} \end{aligned}$$

$$\begin{aligned}
& -\pi\sqrt{\hat{a}}\frac{1-s_\beta^2}{256s_\beta^2}\left[1024-640\hat{a}+56\hat{a}^2+3\hat{a}^3\right] \\
& +\frac{19}{s_\beta^2}-\frac{1-s_\beta^2}{22050s_\beta^2}\left[970200\hat{a}-376075\hat{a}^2+24843\hat{a}^3+6912\hat{a}^4\right]+\mathcal{O}(\hat{a}^5)\Big\}.
\end{aligned} \tag{13}$$

In the limit  $\hat{a} \rightarrow 0$  or  $a \rightarrow \infty$  one obtains

$$\Delta\rho_{1,\text{Higgs}}^{\text{SUSY}} = 3 \frac{G_F^2}{128 \pi^4} m_t^4 \frac{1}{s_\beta^2} \left[19 - 2\pi^2\right] + \mathcal{O}(\hat{a}). \tag{14}$$

## 4 Numerical analysis

### 4.1 The expansion formula

We first analyze the validity of the two expansion formulas, eqs. (11) and (14). In Fig. 4 we show the result for  $\delta_{1,\text{Higgs}}^{\text{SUSY}}$ , defined by

$$\Delta\rho_{1,\text{Higgs}}^{\text{SUSY}} = 3 \frac{G_F^2}{128 \pi^4} m_t^4 \times \delta_{1,\text{Higgs}}^{\text{SUSY}}, \tag{15}$$

as a function of  $b = M_A/m_t (\equiv 1/\sqrt{a})$  for  $\tan\beta = 3$ . The expansion for  $b \ll 1$  is sufficiently accurate nearly up to  $b = 1$ . The other expansion gives accurate results for  $b \gtrsim 2$ . For larger  $\tan\beta$  the expansion becomes better, enlarging the validity region for the large  $M_A$  expansion up to  $b \gtrsim 1$ .

### 4.2 Effects on precision observables

In this section we analyze the numerical effect on the precision observables  $M_W$  and  $\sin^2\theta_{\text{eff}}$ , see eq. (2), induced by the additional contribution to  $\Delta\rho$ . In Fig. 5 the size of the leading  $\mathcal{O}(\alpha^2)$  MSSM corrections, eq. (8), is compared for  $\tan\beta = 3, 40$  with the leading  $\mathcal{O}(\alpha^2)$  contribution in the SM for  $M_H^{\text{SM}} = 0$  [12], with the leading MSSM corrections arising from the  $\tilde{t}/\tilde{b}$  sector at  $\mathcal{O}(\alpha)$  [7], and with the corresponding gluon-exchange contributions of  $\mathcal{O}(\alpha\alpha_s)$  [8] (the  $\mathcal{O}(\alpha\alpha_s)$  gluino-exchange contributions [8], which go to zero for large  $m_{\tilde{g}}$ , have been omitted here). For illustration, the left plot ( $\tan\beta = 3$ ) is shown as a function of  $M_A$ , which affects only the  $\mathcal{O}(\alpha^2)$  MSSM contributions, while the right plot ( $\tan\beta = 40$ ) is given as a function of the common SUSY mass scale in the scalar quark sector,  $M_{\text{SUSY}}$ , which affects only the  $\mathcal{O}(\alpha)$  and  $\mathcal{O}(\alpha\alpha_s)$  MSSM contributions. We have furthermore chosen the case of “maximal mixing” in the scalar top sector, which is realized by setting the off-diagonal term in the  $\tilde{t}$  mass matrix,  $X_t$ , to  $X_t = 2M_{\text{SUSY}}$  and yields the maximal value for  $m_h$  for a given  $\tan\beta$  (see Ref. [19] for details). In the right plot the case of no mixing,  $X_t = 0$ , is also shown. The mixing in the scalar bottom sector has been determined by using



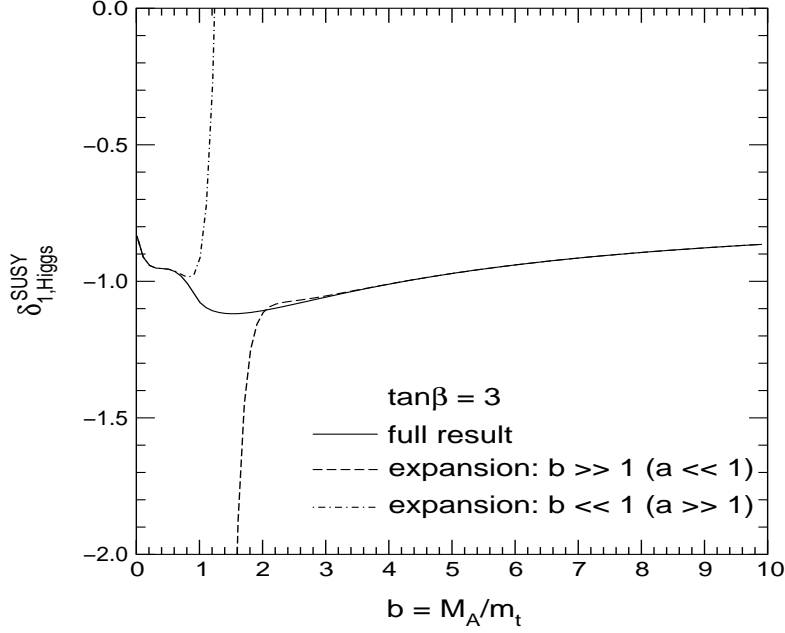


Figure 4: The quality of the expansion formulas, eqs. (11) and (14), is shown as a function of  $b = M_A/m_t (\equiv 1/\sqrt{a})$ .

a bottom quark mass of  $m_b = 4.5$  GeV, and by setting the trilinear couplings to  $A_b = A_t$  and the Higgs mixing parameter to  $\mu = 200$  GeV. The  $\mathcal{O}(\alpha^2)$  contributions in the SM and the MSSM are negative and are for comparison shown with reversed sign.

While for small values of  $M_{SUSY}$  the  $\mathcal{O}(\alpha\alpha_s)$  gluon-exchange contribution in the MSSM is much larger than the  $\mathcal{O}(\alpha^2)$  contribution from eq. (8) (note that in this region of parameter space the approximation of neglecting the scalar-quark contributions in the  $\mathcal{O}(\alpha^2)$  result is no longer valid), they are of approximately equal magnitude for  $M_{SUSY} \approx 500$  GeV (this refers to both the no-mixing and the maximal-mixing case) and compensate each other as they enter with different sign. In this region the two-loop contributions are about one order of magnitude smaller than the  $\mathcal{O}(\alpha)$  MSSM contribution. For  $M_{SUSY} = 1000$  GeV the leading MSSM  $\mathcal{O}(\alpha^2)$  contribution is about three times bigger than the  $\mathcal{O}(\alpha\alpha_s)$  gluon-exchange contribution in the MSSM.

For small  $\tan\beta$  (left plot of Fig. 5) and moderate  $M_A$  ( $M_A \approx 300$  GeV) the new  $\mathcal{O}(\alpha^2)$  MSSM corrections are about two times larger than the leading  $\mathcal{O}(\alpha^2)$  contributions in the SM for  $M_H^{\text{SM}} = 0$ . For large  $M_A$  the decoupling of the extra contributions in the MSSM takes place and the  $\mathcal{O}(\alpha^2)$  MSSM correction approaches the value of the leading  $\mathcal{O}(\alpha^2)$  contributions in the SM for  $M_H^{\text{SM}} = 0$ , as indicated in eqs. (11), (12). For large  $\tan\beta$  (right plot of Fig. 5) the  $\mathcal{O}(\alpha^2)$  MSSM correction and

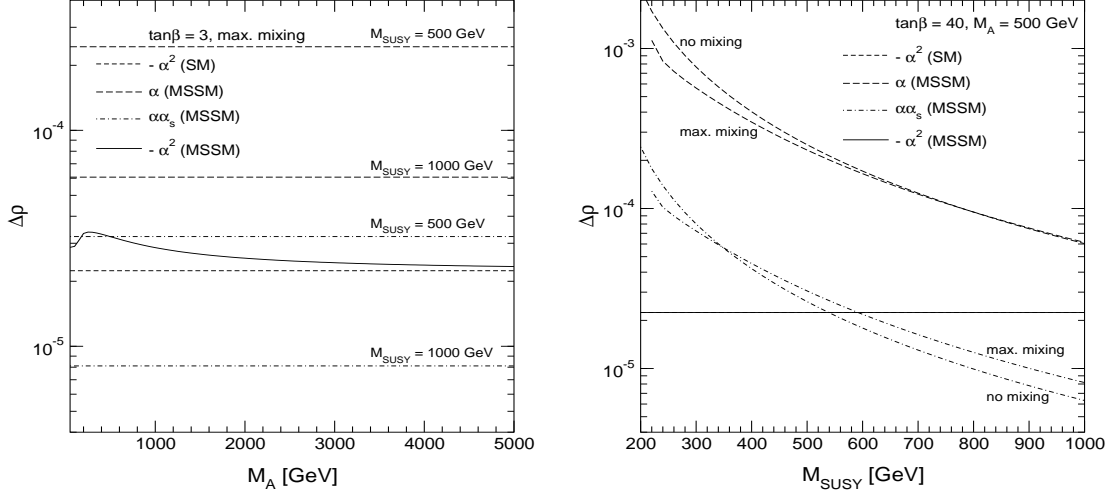


Figure 5: The contribution of the leading  $\mathcal{O}(\alpha^2)$  MSSM corrections,  $\Delta\rho_{1,\text{Higgs}}^{\text{SUSY},\alpha^2}$ , is shown as a function of  $M_A$  for  $\tan\beta = 3$  (left plot) and as a function of  $M_{\text{SUSY}}$  for  $\tan\beta = 40$  (right plot). In the left plot the case of maximal  $\tilde{t}$  mixing is shown, while the right plot displays both the no-mixing and the maximal-mixing case.  $\Delta\rho_{1,\text{Higgs}}^{\text{SUSY},\alpha^2}$  is compared with the leading  $\mathcal{O}(\alpha^2)$  SM contribution with  $M_H^{\text{SM}} = 0$  and with the leading MSSM corrections originating from the  $\tilde{t}/\tilde{b}$  sector of  $\mathcal{O}(\alpha)$  and  $\mathcal{O}(\alpha\alpha_s)$ . Both  $\mathcal{O}(\alpha^2)$  contributions are negative and are for comparison shown with reversed sign.

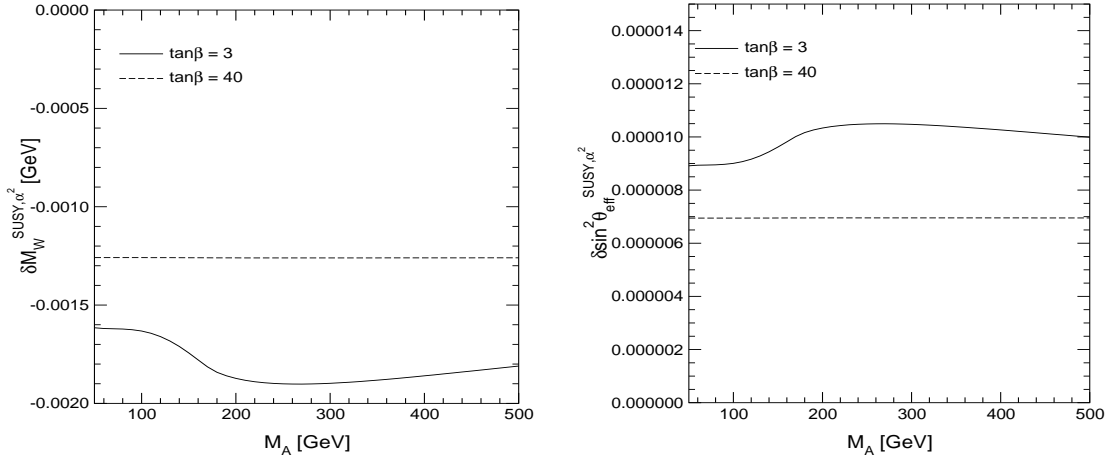


Figure 6: The leading  $\mathcal{O}(\alpha^2)$  MSSM contribution to  $\delta M_W$  (left plot) and  $\delta \sin^2 \theta_{\text{eff}}$  (right plot) is shown as a function of  $M_A$  for  $\tan\beta = 3, 40$ .

the  $\mathcal{O}(\alpha^2)$  contribution in the SM for  $M_H^{\text{SM}} = 0$  are indistinguishable in the plot, in accordance with eq. (10).

It is well known that the  $\mathcal{O}(\alpha^2)$  SM result with  $M_H^{\text{SM}} = 0$  underestimates the result with realistic values of  $M_H^{\text{SM}}$  by about one order of magnitude [18]. One can expect a similar effect in the MSSM once higher order corrections to the Higgs boson sector are properly taken into account, which can enhance  $m_h$  up to  $m_h \lesssim 130$  GeV [19], see Ref. [10].

In Fig. 6 the approximation formulas given in eq. (2) have been employed for determining the shift induced in  $M_W$  and  $\sin^2 \theta_{\text{eff}}$  by the new  $\mathcal{O}(\alpha^2)$  correction to  $\Delta\rho$ . In Fig. 6 the effect for both precision observables is shown as a function of  $M_A$  for  $\tan\beta = 3, 40$ . The effect on  $\delta M_W$  varies between  $-1.5$  MeV and  $-2$  MeV for small  $\tan\beta$  and is almost constant,  $\delta M_W \approx -1.25$  MeV, for  $\tan\beta = 40$ . As above, the constant behavior can be explained by the analytical decoupling of  $\tan\beta$  when  $\tan\beta \gg 1$ , see eq. (10). The induced shift in  $\sin^2 \theta_{\text{eff}}$  lies at or below  $1 \times 10^{-5}$  and shows the same qualitative  $\tan\beta$  dependence as  $\delta M_W$ .

## 5 Conclusions

We have calculated the leading  $\mathcal{O}(G_F^2 m_t^4)$  corrections to  $\Delta\rho$  in the MSSM in the limit of heavy squarks. Short analytical formulas have been obtained for the full result as well as for the cases  $M_A \gg m_t$  and  $M_A \ll m_t$ . As a consistency check we verified that from the MSSM result the corresponding SM result can be obtained in the decoupling limit (i.e.  $M_A \rightarrow \infty$ ).

Numerically we compared the effect of the new contribution with the leading  $\mathcal{O}(\alpha^2)$  SM contribution with  $M_H^{\text{SM}} = 0$  and with the leading MSSM corrections originating from the  $\tilde{t}/\tilde{b}$  sector of  $\mathcal{O}(\alpha)$  and  $\mathcal{O}(\alpha\alpha_s)$ . The numerical effect of the new contribution exceeds the one of the leading QCD correction of  $\mathcal{O}(\alpha\alpha_s)$  in the scalar quark sector for  $M_{\text{SUSY}} \gtrsim 500$  GeV. It is always larger than the leading  $\mathcal{O}(\alpha^2)$  SM contribution with  $M_H^{\text{SM}} = 0$ , reaching approximately twice its value for small  $\tan\beta$  and moderate  $M_A$ .

The numerical effect of the new contribution on the precision observables  $M_W$  and  $\sin^2 \theta_{\text{eff}}$  is relatively small, up to  $-2$  MeV for  $M_W$  and  $+1 \times 10^{-5}$  for  $\sin^2 \theta_{\text{eff}}$ . It should be noted, however, that the  $\mathcal{O}(\alpha^2)$  SM result with  $M_H^{\text{SM}} = 0$ , to which the new result corresponds, underestimates the result with realistic values of  $M_H^{\text{SM}}$  by about one order of magnitude. A similar behavior can also be expected for the MSSM corrections. An extension of our present result to the case of non-zero values of the lightest  $\mathcal{CP}$ -even Higgs boson mass will be undertaken in a forthcoming publication.

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